THE NEW COMMODITY MARKETS: I. INTRODUCTION

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PLAN FOR THE LECTURES

▶ Lecture I: Commodity Markets
  ▶ Production, Transportation, Storage, Delivery
  ▶ Spot / Forward Markets
  ▶ Convenience Yield

▶ Lecture II: Spread Options
  ▶ Why Spread Options
  ▶ Spark Spread Options
  ▶ Real Option Theory Asset Valuation
  ▶ More Asset Valuation
    ▶ Plant Optionality Valuation
    ▶ Financial Valuation
    ▶ Valuing Storage Facilities

▶ Related Markets
  ▶ Weather Markets
  ▶ Emission Markets
Basic Textbooks on the Subject

- F.E. Benth, J.S. Benth, and S. Koekebakker,
  *Stochastic Modeling of Electricity and Related Markets*,

- L. Clewlow, and C. Strickland,
  *Energy Derivatives: Pricing and Risk Management*,
  Lacima Productions, 2000

- A. Eydeland, and K. Wolyniec,
  *Energy and Power Risk Management: New Developments in Modeling, Pricing and Hedging*,
  Wiley, Finance, 2003

- H. Geman,
  *Commodities and commodity derivatives: Modeling and Pricing of Agriculturals, Metals and Energy*,
  Wiley, Finance, 2005

- H. Geman,
  *Risk Management in Commodity Markets: From Shipping to Agriculturals and Energy*,
  Wiley, Finance, 2008

- R. Weron,
  *Modeling and Forecasting Electricity Loads and Prices: a statistical approach*,
  Wiley, Finance, 2007
Pricing by Equilibrium Arguments
- Supply / Demand
- Inventory (Storage / Delivery)
- Convenience yield
- Standard Valuation Methods do not apply
  (e.g. present value of flow of future dividend)

Physical Markets
- Spot (immediate delivery) Markets
- Forward Markets

Volume Explosion with Financially Settled Contracts
- Physical / Financial Contracts
- Exchanges serve as Clearing Houses
- Speculators provide Liquidity

Diversification (believed to be negatively correlated with stocks)
A **given commodity** is traded on **one** (or a small number of) **specialized exchange(s)**

<table>
<thead>
<tr>
<th>Exchange</th>
<th>Location</th>
<th>Contracts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chicago Board of Trade (CBOT)</td>
<td>Chicago</td>
<td>Grains, Ethanol, Metals</td>
</tr>
<tr>
<td>Chicago Mercantile Exch. (CME)</td>
<td>Chicago, US</td>
<td>Meats, Currencies, Eurodollars</td>
</tr>
<tr>
<td>Intercontinental Exch. (ICE)</td>
<td>Atlanta, US</td>
<td>Energy, Emissions, Agricultural</td>
</tr>
<tr>
<td>Kansas City Board of Trade (KCBT)</td>
<td>Kansas City, US</td>
<td>Agricultural</td>
</tr>
<tr>
<td>Climex (CLIMEX)</td>
<td>Amsterdam, NL.</td>
<td>Emissions</td>
</tr>
<tr>
<td>NYSE Liffe</td>
<td>Europe</td>
<td>Agricultural</td>
</tr>
<tr>
<td>European Climate Exch. (ECX)</td>
<td>Europe</td>
<td>Emissions</td>
</tr>
<tr>
<td>London Metal Exch. (LME)</td>
<td>London, UK</td>
<td>Industrial Metals, Plastics</td>
</tr>
</tbody>
</table>
Gaining Exposure to Commodity

- Purchasing Physical Commodity
  - Transportation / Delivery
  - Storage / Perishability
- Purchasing Stock in Commodity Intensive Businesses
  - Indirect exposure
  - Shares of natural resource companies non-perfectly correlated with commodity prices
- Investing in Commodity Futures & Options
  - Transparency & Integrity (clearing)
  - Small initial investment (margin calls)
  - Careful Rolling (e.g. to avoid physical delivery)
- Investing in Commodity Indexes and Commodity Funds
  - Passive investment (no need for a CTA)
  - Can reconstruct *historical* performance
# Original Commodity Indexes

<table>
<thead>
<tr>
<th></th>
<th>CRB/CCI</th>
<th>GSCI</th>
<th>Rogers RMI</th>
<th>DJ-AIG</th>
</tr>
</thead>
<tbody>
<tr>
<td>Started (Year)</td>
<td>11957/986</td>
<td>1992</td>
<td>1998</td>
<td>1999</td>
</tr>
<tr>
<td>Exchange Traded</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Number of Components</td>
<td>17</td>
<td>22</td>
<td>35</td>
<td>20</td>
</tr>
<tr>
<td>Energy</td>
<td>18%</td>
<td>50%</td>
<td>44%</td>
<td>31%</td>
</tr>
<tr>
<td>Metals (Gold)</td>
<td>24 6</td>
<td>12 2</td>
<td>21 3</td>
<td>29 9</td>
</tr>
<tr>
<td>Grains</td>
<td>18</td>
<td>18</td>
<td>21</td>
<td>21</td>
</tr>
<tr>
<td>Food/Fiber</td>
<td>30</td>
<td>10</td>
<td>11</td>
<td>10</td>
</tr>
<tr>
<td>Livestock</td>
<td>12</td>
<td>11</td>
<td>3</td>
<td>9</td>
</tr>
</tbody>
</table>
## Major Commodity Indexes

<table>
<thead>
<tr>
<th>Sector</th>
<th>Commodity</th>
<th>Exchange</th>
<th>Ticker</th>
<th>S&amp;P - GSCI Weights</th>
<th>DJ-UBSCI Weights</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Weights</td>
<td></td>
<td></td>
<td></td>
<td>24</td>
<td>19</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>99.99%</td>
<td>100.00%</td>
</tr>
<tr>
<td>Energy</td>
<td>Oil (Brent crude)</td>
<td>IPE</td>
<td>LO</td>
<td>13.25%</td>
<td></td>
</tr>
<tr>
<td>Energy</td>
<td>Oil (WTI crude)</td>
<td>NYM</td>
<td>CL</td>
<td>37.51%</td>
<td>13.75%</td>
</tr>
<tr>
<td>Energy</td>
<td>Oil (GasOil)</td>
<td>IPE</td>
<td>QS</td>
<td>4.54%</td>
<td></td>
</tr>
<tr>
<td>Energy</td>
<td>Oil (#2 Heating)</td>
<td>NYM</td>
<td>HO</td>
<td>4.19%</td>
<td>3.65%</td>
</tr>
<tr>
<td>Energy</td>
<td>Natural gas</td>
<td>NYM</td>
<td>NG</td>
<td>4.14%</td>
<td>11.89%</td>
</tr>
<tr>
<td>Energy</td>
<td>Oil (RBOB)</td>
<td>NYM</td>
<td>RB</td>
<td>4.75%</td>
<td>3.71%</td>
</tr>
<tr>
<td>Industrial Metals</td>
<td>Aluminum</td>
<td>LME</td>
<td>AH</td>
<td>2.33%</td>
<td>7.00%</td>
</tr>
<tr>
<td>Industrial Metals</td>
<td>Copper</td>
<td>LME</td>
<td>CA</td>
<td>3.22%</td>
<td>7.31%</td>
</tr>
<tr>
<td>Industrial Metals</td>
<td>Lead</td>
<td>LME</td>
<td>PB</td>
<td>0.45%</td>
<td></td>
</tr>
<tr>
<td>Industrial Metals</td>
<td>Nickel</td>
<td>LME</td>
<td>NI</td>
<td>0.78%</td>
<td>2.88%</td>
</tr>
<tr>
<td>Industrial Metals</td>
<td>Zinc</td>
<td>LME</td>
<td>ZS</td>
<td>0.60%</td>
<td>3.14%</td>
</tr>
<tr>
<td>Precious Metals</td>
<td>Gold</td>
<td>CMX</td>
<td>GC</td>
<td>3.01%</td>
<td>7.86%</td>
</tr>
<tr>
<td>Precious Metals</td>
<td>Silver</td>
<td>CMX</td>
<td>SI</td>
<td>0.32%</td>
<td>2.89%</td>
</tr>
</tbody>
</table>
## Major Commodity Indexes (cont.)

<table>
<thead>
<tr>
<th>Sector</th>
<th>Commodity</th>
<th>Exchange</th>
<th>Ticker</th>
<th>S&amp;P - GSCI Weights</th>
<th>DJ-UBSCI Weights</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture</td>
<td>Cocoa</td>
<td>CSC</td>
<td>CC</td>
<td>0.40%</td>
<td></td>
</tr>
<tr>
<td>Agriculture</td>
<td>Coffee &quot;C&quot;</td>
<td>CSC</td>
<td>KC</td>
<td>0.76%</td>
<td>2.97%</td>
</tr>
<tr>
<td>Agriculture</td>
<td>Corn</td>
<td>CBT</td>
<td>C</td>
<td>3.55%</td>
<td>5.72%</td>
</tr>
<tr>
<td>Agriculture</td>
<td>Cotton #2</td>
<td>NYC</td>
<td>CT</td>
<td>1.19%</td>
<td>2.27%</td>
</tr>
<tr>
<td>Agriculture</td>
<td>Wheat (Kansas)</td>
<td>KCBT</td>
<td>KW</td>
<td>0.82%</td>
<td></td>
</tr>
<tr>
<td>Agriculture</td>
<td>Soybean oil</td>
<td>CBT</td>
<td>BO</td>
<td></td>
<td>2.88%</td>
</tr>
<tr>
<td>Agriculture</td>
<td>Soybeans</td>
<td>CBT</td>
<td>S</td>
<td>2.64%</td>
<td>7.60%</td>
</tr>
<tr>
<td>Agriculture</td>
<td>Sugar</td>
<td>CSC</td>
<td>SB</td>
<td>2.33%</td>
<td>2.99%</td>
</tr>
<tr>
<td>Agriculture</td>
<td>Wheat (Chicago)</td>
<td>CBT</td>
<td>W</td>
<td>3.90%</td>
<td>4.80%</td>
</tr>
<tr>
<td>Livestock</td>
<td>Feeder cattle</td>
<td>CME</td>
<td>FC</td>
<td>0.61%</td>
<td></td>
</tr>
<tr>
<td>Livestock</td>
<td>Lean hogs</td>
<td>CME</td>
<td>LH</td>
<td>1.51%</td>
<td>2.40%</td>
</tr>
<tr>
<td>Livestock</td>
<td>Live cattle</td>
<td>CME</td>
<td>LC</td>
<td>3.19%</td>
<td>4.29%</td>
</tr>
</tbody>
</table>
DB Liquidity Commodity Index (DBLCI)

- Launched in 2003
- Equally weighted
- Basis for Index Tracking Funds

<table>
<thead>
<tr>
<th>Index Weight</th>
<th>Contract Months</th>
<th>Exchange</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Energy</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>WTI Crude Oil</td>
<td>35.00%</td>
<td>Jan-Dec</td>
</tr>
<tr>
<td>Heating Oil</td>
<td>20.00%</td>
<td>Jan-Dec</td>
</tr>
<tr>
<td><strong>Precious Metals</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gold</td>
<td>10.00%</td>
<td>Dec</td>
</tr>
<tr>
<td><strong>Industrial Metals</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Aluminium</td>
<td>12.50%</td>
<td>Dec</td>
</tr>
<tr>
<td><strong>Grains</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corn</td>
<td>11.25%</td>
<td>Dec</td>
</tr>
<tr>
<td>Wheat</td>
<td>11.25%</td>
<td>Dec</td>
</tr>
</tbody>
</table>
**Empirical Facts**

- In 2006 - 2007, index fund investment increased **from 90 billion to 200 billion USD** (source: Barclays)
- Simultaneously, **commodity prices increased 71%** as measured by the CRB index
- Prices declined from June 2008 through early 2009

**Possible explanations**

- Large scale speculative buying by index funds created a bubble, (futures prices far exceeded fundamental values)
- Some economists ([Krugman](#) 2008; [Pirrong](#) 2008; [Sanders](#) and [Irwin](#) 2008, [Hamilton](#) 2009, [Kilian](#) 2009) are **skeptic** about the "bubble theory"

  
  "... Prices of commodities are set by supply-demand, rapid growth in emerging economies (e.g. China) increased demand and caused the 2008 surge in price. ...."
Increased participation in futures markets by nontraditional investors deemed disruptive

**Blamed for the 2007-2008 Food Crisis**: "Casino of Hunger: How Wall Street Speculators Fueled the Global Food Crisis"

A report from **U.S. Senate Permanent Subcommittee on Investigation**

"... finds that there is significant and persuasive evidence to conclude that these commodity index traders, in the aggregate, were one of the major causes of unwarranted changes here increases in the price of wheat futures contracts relative to the price of wheat in the cash market....."

**48 Agriculture Ministers** meeting in Berlin said there were

"... concerned that excessive price volatility and speculation on international agricultural markets might constitute a threat to food security, according to a joint statement handed out to reporters on Jan. 22, 2011...."
Empirical Facts

- Commodity Index trading tightened correlations between commodities (Tang-Xiong 2010)
- Scale dependent phenomenon: Do high frequency traders see these correlation increases?

Financialization of Commodities: two talks during this workshop

- Wei Xiong
- Ronnie Sircar
Are Commodities Uncorrelated with Equities?

Time Series Plot of BETA.ts

Instantaneous Dependence ($\beta$) of GSCI-TR returns upon S&P 500 returns
**First Challenge: Constructing Forward Curves**

- **How can it be a challenge?**
  - Just do a PCA!
    - "OK" for Crude Oil (backwardation/contango → 3 factors)
    - Not settled for Gas
    - Does not work for Electricity
  - Extreme complexity & size of the data (location, grade, peak/off peak, firm/non firm, interruptible, swings, etc)
  - Incomplete and inconsistent sources of information
  - Liquidity and wide Bid-Ask spreads (smoothing)
  - Length of the curve (extrapolation)

- **Dynamic models à la HJM:**
CRUDE OIL

Time Series Plot of CO.ts
CRUDE OIL FORWARD SURFACE
Early Forward Curves
MORE CRUDE OIL FORWARD CURVES

Time To Maturity

Graph showing various crude oil forward curves with different maturity times.
Is the Forward the Expected Value of Future Spots?
CRUDE OIL FORWARD CURVES 01/03/1995 – 12/31/1998
CRUDE OIL FORWARD CURVES 01/02/2006 TO 12/31/2010
**Spot Forward Relationship**

In financial models where one can hold positions at no cost

\[ F(t, T) = S(t)e^{r(T-t)} \]

by a simple **cash & carry arbitrage** argument. In particular

\[ F(t, T) = \mathbb{E}\{S(T) \mid \mathcal{F}_t\} \]

for risk neutral expectations.

**Perfect Price Discovery**

In general (theory of normal **backwardation**)

- \(F(t, T)\) is a **downward biased** estimate of \(S(T)\)
- Spot price exceeds the forward prices
**Notion of Convenience Yield**

**Forward Price** = (risk neutral) conditional expectation of future values of **Spot Price**

- No **cash & carry** arbitrage argument
  - Is the spot really tradable?
  - What are its dynamics?
  - How do we *risk-adjust* them?

- **Convenience Yield** for storable commodities
  - Natural Gas, Crude Oil, . . .
  - Correct interest rate to compute present values
  - Does not apply to Electricity
**SPOT-FORWARD RELATIONSHIP FOR COMMODITIES**

For **storable** commodities (still same **cash & carry arbitrage** argument)

\[
F(t, T) = S(t)e^{(r-\delta)(T-t)}
\]

for \( \delta \geq 0 \) called **convenience yield**. **(NOT FOR ELECTRICITY !)**

Decompose \( \delta = \delta_1 - c \) with

- \( \delta_1 \) benefit from owning the physical commodity
- \( c \) cost of storage

Then

\[
f(t, T) = e^{r(T-t)} e^{-\delta_1(T-t)} e^{-c(T-t)}
\]

- \( e^{r(T-t)} \) cost of **financing** the purchase
- \( e^{c(T-t)} \) cost of **storage**
- \( e^{-\delta_1(T-t)} \) sheer **benefit from owning** the physical commodity
Backwardation / Contango Duality

Backwardation

- $T \leftrightarrow F(t, T) = S(t)e^{(r+c-\delta_1)(T-t)}$ decreasing if $r + c < \delta_1$
  - Low cost of storage
  - Low interest rate
  - High benefit in holding the commodity

Contango

- $T \leftrightarrow F(t, T) = S(t)e^{(r+c-\delta_1)(T-t)}$ increasing if $r + c \geq \delta_1$
NATURAL GAS

Time Series Plot of NG.ts

[Graph showing a time series plot with data points from 1995 to 2010.]
NG Forward Curves 01/03/1995 – 12/31/1998
NG Forward Curves 01/02/2006 to 12/31/2010
**Commodity Convenience Yield Models**

**Gibson-Schwartz** Two-factor model
- $S_t$ commodity spot price
- $\delta_t$ convenience yield

**Risk Neutral Dynamics**

\[
\begin{align*}
\text{d}S_t &= (r_t - \delta_t)S_t \text{d}t + \sigma S_t \text{d}W^1_t,
\text{d}\delta_t &= \kappa(\theta - \delta_t) \text{d}t + \sigma_\delta \text{d}W^2_t
\end{align*}
\]

**Major Problems**
- Explicit formulae (exponential affine model)
- Convenience yield implied from forward contract prices
- Unstable & Inconsistent (R.C.-M. Ludkovski)
Lack of Consistency

Exponential Affine Model

\[
F(t, T) = S_t e^{\int_t^T r_s ds} e^{B(t, T) \delta_t + A(t, T)}
\]

where

\[
B(t, T) = \frac{e^{-\kappa(T-t)} - 1}{\kappa},
\]

\[
A(t, T) = \frac{\kappa \theta + \rho \sigma_s \gamma}{\kappa^2} \left(1 - e^{-\kappa(T-t)} - \kappa(T - t)\right) + \frac{\gamma^2}{\kappa^3} \left(2\kappa(T - t) - 3 + 4e^{-\kappa(T-t)} - e^{-2\kappa(T-t)}\right).
\]

- For each \( T \), one can imply \( \delta_t \) from \( F(t, T) \)
- Inconsistency in the implied \( \delta_t \)
- Ignores **Maturity Specific** effects
Crude Oil convenience yield implied by a 3 month futures contract (left)
Difference in implied convenience yields between 3 and 12 month contracts.
Convenience Yield Models Revisited

Use **forward** $F_t = F(t, T_0)$ instead of **spot** $S_t$ ($T_0$ fixed maturity)

Historical Dynamics

\[
\begin{align*}
    dF_t &= (\mu_t - \delta_t)F_t \, dt + \sigma F_t \, dW^1_t, \\
    d\delta_t &= \kappa(\theta - \delta_t) dt + \sigma_\delta \, dW^2_t
\end{align*}
\]

or more generally

\[
d\delta_t = b(\delta_t, F_t) dt + \sigma_\delta(\delta_t, F_t) dW^2_t
\]

We assume

- $F_t$ is **tradable** (hence **observable**)
- (Forward) convenience yield $\delta_t$ **not observable** (filtering)

Different from Bjork-Landen’s Risk Neutral Term Structure of Convenience Yield
**The Case of Power**

**Several obstructions**
- Cannot store the physical commodity
- Delivery **over** a period \([T_1, T_2]\) (Benth)
- Does the forward price converge as the time to maturity goes to 0?

**Mathematical spot?**

\[
S(t) = \lim_{T \downarrow t} F(t, T)
\]

**Sparse Forward Data**
- Lack of **transparency** (manipulated indexes)
- Poor (or lack of) **reporting** by fear of law suits
- **CCRO** white paper(s)
Dynamic Model for Forward Curves

$n$-factor forward curve model

\[
\frac{dF(t, T)}{F(t, T)} = \mu(t, T)dt + \sum_{k=1}^{n} \sigma_k(t, T)dW_k(t) \quad t \leq T
\]

- \( W = (W_1, \ldots, W_n) \) is a \( n \)-dimensional standard Brownian motion,
- drift \( \mu \) and volatilities \( \sigma_k \) are deterministic functions of \( t \) and time-of-maturity \( T \)
- \( \mu(t, T) \equiv 0 \) for pricing
- \( \mu(t, T) \) calibrated to historical data for risk management
Explicit Solution

\[ F(t, T) = F(0, T) \exp \left[ \int_0^t \left[ \mu(s, T) - \frac{1}{2} \sum_{k=1}^n \sigma_k(s, T)^2 \right] ds + \sum_{k=1}^n \int_0^t \sigma_k(s, T) dW_k(s) \right] \]

Forward prices are **log-normal** (deterministic coefficients)

\[ F(t, T) = \alpha e^{\beta X - \beta^2 / 2} \]

with \( X \sim N(0, 1) \) and

\[ \alpha = F(0, T) \exp \left[ \int_0^t \mu(s, T) ds \right], \quad \text{and} \quad \beta = \sqrt{\sum_{k=1}^n \int_0^t \sigma_k(s, T)^2 ds} \]
**Dynamics of the Spot Price**

**Spot price** left hand of forward curve

\[ S(t) = F(t, t) \]

We get

\[ S(t) = F(0, t) \exp \left[ \int_{0}^{t} \left[ \mu(s, t) - \frac{1}{2} \sum_{k=1}^{n} \sigma_k(s, t)^2 \right] ds + \sum_{k=1}^{n} \int_{0}^{t} \sigma_k(s, t) dW_k(s) \right] \]

and differentiating both sides we get:

\[
dS(t) = S(t) \left[ \left( \frac{1}{F(0, t)} \frac{\partial F(0, t)}{\partial t} + \mu(t, t) + \int_{0}^{t} \frac{\partial \mu(s, t)}{\partial t} ds - \frac{1}{2} \sigma_S(t)^2 \right) \right. \\
- \sum_{k=1}^{n} \int_{0}^{t} \sigma_k(s, t) \frac{\partial \sigma_k(s, t)}{\partial t} ds + \left. \sum_{k=1}^{n} \int_{0}^{t} \frac{\partial \sigma_k(s, t)}{\partial t} dW_k(s) \right] dt + \sum_{k=1}^{n} \sigma_k(t, t) dW_k(t) \]

**Spot volatility**

\[
\sigma_s(t)^2 = \sum_{k=1}^{n} \sigma_k(t, t)^2. \tag{1}
\]
Clewlow - Strickland

Hence

\[
\frac{dS(t)}{S(t)} = \left[ \frac{\partial \log F(0, t)}{\partial t} + D(t) \right] dt + \sum_{k=1}^{n} \sigma_k(t, t) dW_k(t)
\]

with drift

\[
D(t) = \mu(t, t) - \frac{1}{2} \sigma_S(t)^2 + \int_0^t \frac{\partial \mu(s, t)}{\partial t} ds - \sum_{k=1}^{n} \int_0^t \sigma_k(s, t) \frac{\partial \sigma_k(s, t)}{\partial t} ds
\]

\[
+ \sum_{k=1}^{n} \int_0^t \frac{\partial \sigma_k(s, t)}{\partial t} dW_k(s)
\]
Remarks

Still Clewlow - Strickland

- Interpretation of drift (in a risk-neutral setting)
  - logarithmic derivative of the forward can be interpreted as a discount rate (i.e., the running interest rate)
  - $D(t)$ can be interpreted as a convenience yield

- Drift generally **not** Markovian

- Particular case $n = 1$, $\mu(t, T) \equiv 0$, $\sigma_1(t, T) = \sigma e^{-\lambda(T-t)}$

\[
D(t) = \lambda [\log F(0, t) - \log S(t)] + \frac{\sigma^2}{4} (1 - e^{-2\lambda t})
\]

\[
\frac{dS(t)}{S(t)} = [\mu(t) - \lambda \log S(t)] dt + \sigma dW(t)
\]

exponential OU
**Changing Variables**

\[
\text{time-of-maturity } T \quad \Rightarrow \quad \text{time-to-maturity } \tau
\]

changes dependence upon \( t \)

\[
t \hookrightarrow F(t, T) = F(t, t + \tau) = \tilde{F}(t, \tau)
\]

**Fixed Domain** \([0, \infty)\) for \( \tau \hookrightarrow \tilde{F}(t(\tau)) \)
HEATING OIL FORWARD SURFACE
HO Loadings on their Importance Scale
HH Loadings on their Absolute Importance Scale
CHANGING VARIABLES

time-of-maturity $T \Rightarrow$ time-to-maturity $\tau$

changes dependence upon $t$

$$t \leftrightarrow F(t, T) = F(t, t + \tau) = \tilde{F}(t, \tau)$$

For pricing purposes

- For $T$ fixed, $\{F(t, T)\}_{0 \leq t \leq T}$ is a martingale
- For $\tau$ fixed, $\{\tilde{F}(t, \tau)\}_{0 \leq t}$ is NOT a martingale

$\tilde{F}(t, \tau) = F(t, t + \tau), \quad \tilde{\mu}(t, \tau) = \mu(t, t + \tau), \quad$ and $\tilde{\sigma}_k(t, \tau) = \sigma_k(t, t + \tau),$

In general dynamics become

$$d\tilde{F}(t, \tau) = \tilde{F}(t, \tau) \left[ \left( \tilde{\mu}(t, \tau) + \frac{\partial}{\partial \tau} \log \tilde{F}(t, \tau) \right) dt + \sum_{k=1}^{n} \tilde{\sigma}_k(t, \tau) dW_k(t) \right] ,$$
PCA WITH SEASONALITY

Fundamental Assumption

\[
\sigma_k(t, T) = \sigma(t)\sigma_k(T - t) = \sigma(t)\sigma_k(\tau)
\]

for some function \( t \mapsto \sigma(t) \)

Notice

\[
\sigma_S(t) = \tilde{\sigma}(0)\sigma(t)
\]

provided we set:

\[
\tilde{\sigma}(\tau) = \sqrt{\sum_{k=1}^{n} \sigma_k(\tau)^2}.
\]

Conclusion

\( t \mapsto \sigma(t) \) is (up to a constant) the **instantaneous spot volatility**
Rationale for a New PCA

- Fix times-to-maturity $\tau_1, \tau_2, \ldots, \tau_N$
- Assume on each day $t$, quotes for the forward prices with times-of-maturity $T_1 = t + \tau_1$, $T_2 = t + \tau_2$, $\ldots$, $T_N = t + \tau_N$ are available

$$\frac{d\tilde{F}(t, \tau_i)}{\tilde{F}(t, \tau_i)} = \left(\tilde{\mu}(t, \tau_i) + \frac{\partial}{\partial \tau} \log \tilde{F}(t, \tau_i)\right) dt + \sigma(t) \sum_{k=1}^{n} \sigma_k(\tau_i) dW_k(t) \quad i = 1, \ldots, N$$

Define $\mathbf{F} = [\sigma_k(\tau_i)]_{i=1,\ldots,N, \ k=1,\ldots,n}$.

$$d \log \tilde{F}(t, \tau_i) = \left(\tilde{\mu}(t, \tau_i) + \frac{\partial}{\partial \tau_i} \log \tilde{F}(t, \tau_i) - \frac{1}{2} \sigma(t)^2 \tilde{\sigma}(\tau_i)^2\right) dt + \sigma(t) \sum_{k=1}^{n} \sigma_k(\tau_i) dW_k(t)$$

Instantaneous variance/covariance matrix $\{M(t); t \geq 0\}$ defined by:

$$d[\log \tilde{F}(\cdot, \tau_i), \log \tilde{F}(\cdot, \tau_j)]_t = M_{i,j}(t) dt$$

satisfies

$$M(t) = \sigma(t)^2 \left(\sum_{k=1}^{n} \sigma_k(\tau_i) \sigma_k(\tau_j)\right)$$

or equivalently

$$M(t) = \sigma(t)^2 \mathbf{F} \mathbf{F}^*$$
Strategy Summary

- Estimate instantaneous spot volatility $\sigma(t)$ (in a rolling window)
- Estimate $\mathbf{F}^*$ from historical data as the empirical auto-covariance of $\ln(F(t, \cdot)) - \ln(F(t-1, \cdot))$ after normalization by $\sigma(t)$
- Instantaneous auto-covariance structure of the entire forward curve becomes time independent
- Do SVD of auto-covariance matrix and get
  \[ \tau \leftrightarrow \sigma_k(\tau) \]
- Choose order $n$ of the model from their relative sizes
THE CASE OF NATURAL GAS

Instantaneous standard deviation of the Henry Hub natural gas spot price computed in a sliding window of length 30 days.
HH De-Seasonalized Loadings on their Absolute Importance Scale
Mean Reversion toward the cost of production
The example of the power prices

- Reduced Form Models
  - Nonlinear effects (exponential $OU^2$)
Mean Reversion toward the cost of production
The example of the power prices

- **Reduced Form Models**
  - Nonlinear effects (exponential $OU^2$)
  - Jumps (Geman-Roncoroni, Benth, Cartea, Meyer-Brandis, ...)

- **Structural Models**
  - Inelastic Demand
  - The Supply Stack

**Barlow** (based on merit order graph)
- $s_t(x)$ supply at time $t$ when power price is $x$
- $d_t(x)$ demand at time $t$ when power price is $x$

**Power price** at time $t$ is number $S_t$ such that

$$s(S_t) = d_t(S_t)$$
Example of a merit graph (Alberta Power Pool, courtesy M. Barlow)
BARLOW’S PROPOSAL

\[ S(t) = \begin{cases} 
  f_\alpha(X_t) & 1 + \alpha X_t > \epsilon_0 \\
  \epsilon_0^{1/\alpha} & 1 + \alpha X_t \leq \epsilon_0 
\end{cases} \]

for the non-linear function

\[ f_\alpha(x) = \begin{cases} 
  (1 + \alpha x)^{1/\alpha}, & \alpha \neq 0 \\
  e^x & \alpha = 0 
\end{cases} \]

of an OU diffusion

\[ dX_t = -\lambda(X_t - \bar{x})dt + \sigma dW_t \]
Monte Carlo Sample from Barlow’s Spot Model (courtesy M. Barlow)
Example of a Monte Carlo Sample from the Exponential of an $OU^2$
Consider the case of **PJM** (Pennsylvania - New Jersey - Maryland)

- Over 3,000 nodes in the transmission network
- Each day, and for each node
  - Real time prices
  - Day-ahead prices
  - Hour by hour load prediction for the following day
- **Historical prices**
  - In 2003 over 100,000 instances of **NEGATIVE PRICES**
    - Geographic clusters
    - Time of the year (**shoulder months**)
    - Time of the day (**night**)
- **Possible Explanations**
  - Load miss-predicted
  - High temperature volatility
For many contracts, delivery needs to match demand

- **Demand** for energy highly correlated with **temperature**
  - Heating Season (winter) HDD
  - Cooling Season (summer) CDD
- **Stylized Facts** and **First (naive) Models**
  - Electricity demand = $\beta \cdot \text{weather} + \alpha$
Daily Load versus Daily Temperature (PJM)
For many contracts, delivery needs to match demand

- **Demand** for energy highly correlated with temperature
  - Heating Season (winter) HDD
  - Cooling Season (summer) CDD

- **Stylized Facts** and First (naive) Models
  - Electricity demand = $\beta \times \text{weather} + \alpha$
    - Not true all the time
    - Time dependent $\beta$ by filtering!
  - From the stack: Correlation (Gas,Power) = f(weather)
    - No significance, too unstable
    - Could it be because of heavy tails?

- **Weather dynamics** need to be included
  - Another Source of Incompleteness
In 2001, PU budget for electricity was 2.8 M $ in the red! (PU is small)

- Never Again such a Short Fall !!!
- Student (Greg Larkin) Senior Thesis
- Hedging Volume Risk
  - Protection against the Weather Exposure
  - Temperature Options on CDDs (Extreme Load)
- Hedging Volume & Basis Risk
  - Protection against Gas & Electricity Price Spikes
  - Gas purchase with Swing Options
**MITIGATING VOLUME RISK WITH SWING OPTIONS**

Exposure to spikes in prices of
- Natural Gas (used to fuel the plant)
- Electricity Spot (in case of overload)

**Proposed Solution**
- Forward Contracts
- Swing Options

*Pretty standard*
Use **Swing Options**

Multiple Rights to deviate (within bounds) from base load contract level

**Pricing & Hedging** quite involved!

- Tree/Forest Based Methods
  - Direct Backward Dynamic Programing Induction (à la Jaillet-Ronn-Tompaidis)

**New Monte Carlo Methods**

- Nonparametric Regression (à la Longstaff-Schwarz) Backward Dynamic Programing Induction
Review: Classical Optimal Stopping Problem: American Option

- $X_0, X_1, X_2, \ldots, X_n, \ldots$ rewards
- Right to ONE Exercise
- Mathematical Problem

\[
\sup_{0 \leq \tau \leq T} \mathbb{E}\{X_\tau\}
\]

Mathematical Solution

- Snell’s Envelop
- Backward Dynamic Programming Induction in Markovian Case

Standard, Well Understood
NEW MATHEMATICAL CHALLENGES

In its simplest form the problem of Swing/Recall option pricing is an **Optimal Multiple Stopping Problem**

- $X_0, X_1, X_2, \ldots, X_n, \ldots$ rewards
- Right to $N$ Exercises
- Mathematical Problem

$$\sup_{0 \leq \tau_1 < \tau_2 < \cdots < \tau_N \leq T} \mathbb{E}\{X_{\tau_1} + X_{\tau_2} + \cdots + X_{\tau_N}\}$$

- **Refraction** period $\theta$

$$\tau_1 + \theta < \tau_2 < \tau_2 + \theta < \tau_3 < \cdots < \tau_{N-1} + \theta < \tau_N$$

Part of recall contracts & crucial for continuous time models
Instruments with Multiple American Exercises

- Ubiquitous in Energy Sector
  - Swing / Recall contracts
  - End user contracts (EDF)

- Present in other contexts
  - Fixed income markets (e.g. chooser swaps)
  - Executive option programs
    - Reload $\rightarrow$ Multiple exercise, Vesting $\rightarrow$ Refraction, $\cdots$
  - Fleet Purchase (airplanes, cars, $\cdots$)

- Challenges
  - Valuation
  - Optimal exercise policies
  - Hedging
Some Mathematical Problems

Recursive re-formulation into a hierarchy of classical optimal stopping problems

- Development of a theory of Generalized Snell's Envelop in continuous time setting
- Find a form of Backward Dynamic Programing Induction in Markovian Case
- Design & implement efficient numerical algorithms for finite horizon case

Results

- Perpetual case: abstract nonsense & characterization of the optimal policies
  - R.C. & S.Dayanik (diffusion), R.C. & N.Touzi (GBM)
- Finite horizon case
  - Jaillet - Ronn - Tomapidis (Tree) R.C. N.Touzi (GBM) B.Hambly (chooser swap)
Exercise regions for $N = 5$ rights and finite maturity computed by Malliavin-Monte-Carlo.
Mitigation of Volume Risk with Temperature Options

- Rigorous Analysis of the Dependence between the Budget Shortfall and Temperature in Princeton
- Use of Historical Data (sparse) & Define of a Temperature Protection
  - Period of the Coverage
  - Form of the Coverage
- Search for the Nearest Weather Stations with HDD/CDD Trades
  - La Guardia Airport (LGA)
  - Philadelphia (PHL)
- Define a Portfolio of LGA & PHL forward / option Contracts
- Construct a LGA / PHL basket
Pricing: How Much is it Worth to PU?

- Actuarial / Historical Approach
  - Burn Analysis
  - Temperature Modeling & Monte Carlo VaR Computations
  - Not Enough Reliable Load Data
- Expected (Exponential) Utility Maximization (A. Danilova)
  - Use Gas & Power Contracts
  - Hedging in Incomplete Models
  - Indifference Pricing
  - Very Difficult Numerics (whether PDE’s or Monte Carlo)
The Weather Markets

Weather is an essential economic factor

- ‘Weather is not just an environmental issue; it is a major economic factor. At least 1 trillion USD of our economy is weather-sensitive’ (William Daley, 1998, US Commerce Secretary)

- 20% of the world economy is estimated to be affected by weather

- Energy and other industrial sectors, Entertainment and Tourism Industry, ...

- WRMA

Weather Derivatives as a Risk Transfer Mechanism (El Karoui - Barrieu)
Total Notional Value of weather contracts: (in million USD) Price Waterhouse Coopers market survey.
Weather Derivatives

- **OTC** Customer tailored transactions
  - Temperature, Precipitation, Wind, Snow Fall, ..... 
- **CME** (≈ 50%) (Temperature - Launched in 1999)
  - 18 American cities
  - 2 Japanese cities (Tokyo and Osaka)
AN EXAMPLE OF PRECIPITATION CONTRACT

▶ Physical Underlying Daily Index:
  ▶ Precipitation in Paris
  ▶ A day is a rainy day if precipitation exceeds 2mm

▶ Season
  ▶ 2000: April thru August + September weekends
  ▶ 2001: April thru August + September weekends
  ▶ 2002: April thru August + September weekends

▶ Aggregate Index
  ▶ Total Number of Rainy Days in the Season

▶ Pay-Off
  ▶ Strike, Cap, Rate
RAINFALL OPTION CONTINUED

- Who Wanted this Deal?
  - A Natural trying to hedge RainFall Exposure (Asterix Amusement Park)

- Who was willing to take the other side?
  - Speculators
  - Insurance Companies
  - Re-insurance Companies
  - Statistical Arbitrageurs
  - Investment Banks
  - Hedge Funds
  - Endowment Funds
  - .................
Other Example: Precipitation / Snow Pack

- City of Sacramento
  - HydroPower Electricity
- Who was on the other side?
  - Large Energy Companies (*Aquila, Enron*)

Who is covering for them?
Jargon of Temperature Options

For a given location, on any given day $t$

$$CDD_t = \max\{T_t - 65, 0\} \quad HDD_t = \max\{65 - T_t, 0\}$$

Season

- One Month (CME Contracts)
- May 1st September 30 (CDD season)
- November 1st March 31st (HDD season)

Index

- Aggregate number of DD in the season

$$I = \sum_{t \in \text{Season}} CDD_t \quad \text{or} \quad I = \sum_{t \in \text{Season}} HDD_t$$

Pay-Off

- Strike $K$, Cap $C$, Rate $\alpha$
Pay-off = \min\{\max\{\alpha \times (I - K), 0\}, C\}

\xi = f(DD)
**Put with a Floor**

\[ \xi = f(DD) \]

Pay-off = \( \min \{ \max \{ \alpha \ast (K - I), 0 \}, C \} \)
CO LLAR

\[ \xi = f(DD) \]

Diagram showing a function of \( DD \) with points labeled \( K_p \) and \( K_c \).
Famous Example of Weather Station Change in Charlotte (NC).
STYLIZED SPREADSHEET OF A BASKET OPTION

- **Structure**: Heating Degree Day (HDD) Floor (Put)
- **Index**: Cumulative HDDs
- **Term**: November 1, 2007  February 28, 2008
- **Stations**:
  - New York, LaGuardia  57.20%
  - Boston, MA  24.5%
  - Philadelphia, PA  12.00%
  - Baltimore, MD  6.30%
- **Floor Strike**: 3130 HDDs
- **Payout**: USD 35,000/HDD
- **Limit**: USD 12,500,000
- **Premium**: USD 2,925,000
WEATHER AND COMMODITY

- **Stand-alone**
  - temperature ($\approx 80\%$)
  - precipitation ($\approx 10\%$)
  - wind ($\approx 5\%$)
  - snow fall ($\approx 5\%$)

- **In-Combination**
  - natural gas
  - power
  - heating oil
  - propane

- Agricultural risk (yield, revenue, input hedges and trading)
- Power outage - contingent power price options
WEATHER (TEMPERATURES) DERIVATIVES

- Still Extremely **Illiquid** Markets (except for **front month**)
- **Misconception**: Weather Derivative = Insurance Contract
  - No secondary market (Except on **Enron-on-Line!!!**)
- **Mark-to-Market** (or Model)
  - Essentially never changes
  - At least, Not Until Meteorology **kicks in** (10-15 days before maturity)
  - Then Mark-to-Market (or Model) **changes** every day
  - Contracts change hands
  - That’s when major losses occur and money is made
- This **hot period** is not considered in academic studies
  - Need for **updates**: new information coming in (temperatures, forecasts, ....)
  - Filtering is (again) the solution
Daily Average Temperature at La Guardia.
Prediction on 6/1/2001 of daily temperature over the next four months.
The Future of the Weather Markets

- **Social function** of the weather market
  - Existence of a Market of Professionals (for weather risk transfer)
- **Under attack** from
  - (Re-)Insurance industry (but *high frequency / low cost*)
  - Utilities (trying to pass weather risk to end-customer)
    - EDF program in France
    - Weather Normalization Agreements in US
- **Cross Commodity Products**
  - Gas & Power contracts with *weather triggers/contingencies*
  - New (major) players: **Hedge Funds** provide liquidity
- **World Bank**
  - Use weather derivatives instead of insurance contracts
The Weather Market Today

- **Insurance Companies**: Swiss Re, XL, Munich Re, Ren Re
- **Financial Houses**: Goldman Sachs, Deutsche Bank, Merrill Lynch, SocGen, ABN AMRO
- **Hedge funds**: D. E. Shaw, Tudor, Susquehanna, Centaurus, Wolverine

Where is Trading Taking Place?

- **Exchange**: CME (Chicago Mercantile Exchange) 29 cites globally traded, monthly / seasonal contracts
- **OTC**
- Strong end-user demand within the energy sector
INCOMPLETE MARKET MODEL & INDIFFERENCE PRICING

- Temperature Options: Actuarial/Statistical Approach
- Temperature Options: Diffusion Models (Danilova)
- Precipitation Options: Markov Models (Diko)
  - Problem: Pricing in an Incomplete Market
  - Solution: Indifference Pricing à la Davis

\[
\begin{align*}
d\theta_t &= p(t, \theta)dt + q(t, \theta)dW_t^{(\theta)} + r(t, \theta)dQ_t^{(\theta)} \\
\theta_t \text{ non-tradable} \\
S_t \text{ tradable}
\end{align*}
\]
Example: Exponential Utility Function

\[ \tilde{p}_t = \frac{\mathbb{E}\{\tilde{\phi}(Y_T) e^{-\int_t^T V(s,Y_s) ds}\}}{\mathbb{E}\{e^{-\int_t^T V(s,Y_s) ds}\}} \]

where

\[ \tilde{\phi} = e^{-\gamma(1-\rho^2) f} \]
where \( f(\theta_T) \) is the pay-off function of the European call on the temperature

\[ \tilde{p}_t = e^{-\gamma(1-\rho^2) p_t} \]
where \( p_t \) is price of the option at time \( t \)

\[ Y_t \] is the diffusion:

\[ dY_t = [g(t, Y_t) - \frac{\mu(t, Y_t) - r}{\sigma(t, Y_t)} h(t, Y_t)] dt + h(t, Y_t) d\tilde{W}_t \]

starting from \( Y_0 = y \)

\[ V(t, y) = -\frac{1 - \rho^2}{2} \frac{(\mu(t, y) - r)^2}{\sigma(t, y)^2} \]
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