Monte Carlo helps with pricing

René Carmona and **Dario Villani** show how weather-contingent options can be priced using Monte Carlo simulations

Weather-contingent options – contracts that pay out when a weather parameter and another variable reach certain predetermined levels – are present in most commodity markets[1] for the following reasons. First, they offer an inexpensive way of having long volatility positions in commodity markets where investors are scared of prices gapping up. This is because there is a lower probability of the option paying out. Second, they transfer liquidity from a more efficient market to a less efficient one. This is the case for the natural gas and weather markets, which we consider in this article.

Despite these two significant reasons, the market for weather-contingent options is still far from being liquid. This can be attributed to the lack of sound evaluation methods that could help with hedging on the side of market makers and speculators, and with price discovery for end-users.

Our working hypothesis is that the temperature T and the commodity price G are stochastic variables driven by two Wiener processes $W^{(T)}$ and $W^{(G)}$, respectively. That is,

$$T = T(\vec{\tau}, \sigma^{(T)} W^{(T)})$$

and

$$G = G(\vec{\gamma}, \sigma^{(G)} W^{(G)}).$$

$\vec{\tau}$ and $\vec{\gamma}$ are parameters specific to a model dynamics. $\sigma^{(T)}$ is the volatility of the temperature. $\sigma^{(G)}$ is the volatility of the commodity price. The Wiener processes are correlated in the sense that they satisfy

$$dW_t^{(T)} dW_t^{(G)} = \rho dt.$$

The pricing scheme requires the temperature to be modelled with a stochastic process.[2] Burn-cost methods cannot be formulated in terms of Wiener processes[3]; so in order to introduce correlations in a meaningful way, we need to reach the same level of microscopic description usually used for commodity prices.

It is worth pointing out that the commodity price dependence on the temperature is restricted in the present analysis. Indeed, the source of statistical correlation between the two stochastic variables T and G is limited to the noise terms driving the individual dynamics. The components of the vector $\vec{\gamma}$ could be functions of the temperature and so could be the volatility $\sigma^{(G)}$. A theoretical analysis of these models is possible, but the level of mathematical sophistication needed is such that the results lose their intuitive appeal. The interested reader is referred to reference 3 for details and further references.

Every day, the markets fix the value of the parameters $\vec{\tau}$, $\sigma^{(T)}$, $\vec{\gamma}$ and $\sigma^{(G)}$ for weather and the commodity separately. These values are usually obtained by proprietary blends of statistical estimation procedures applied to historical data, and calibration techniques applied to the prices of the actively traded financial instruments (eg swaps or options). In this paper we assume that we have chosen our favourite method to risk adjust separately the temperature and commodity price dynamics by calibration to the traded instruments. Then, we show how to combine these two calibrated univariate models into a bivariate model appropriate for the pricing of a double-trigger weather vs natural gas call option. This will obviously involve the correlation coefficient ρ.

We consider the case of a double-trigger weather vs natural gas call option being priced on 11 April 2003 (some of the data values are fabricated for the sake of simplicity). At the end of the contract period (1 August 2003 through 31 August 2003), the seller pays the buyer for each day the average temperature T_{avg} in New York is above T_K = 84°F and the Daily Gas Daily Index (DGDI) exceeds the Monthly Gas Daily Index (MGDI).

The payout Π is obtained as the sum of the daily values max{DGDI–MGDI,0} multiplied by the volume V. The volume is typically 10,000 million BTU. On any given day, T_{avg} is the semi-sum of the high and the low for the day. The weather station is LaGuardia International Airport. DGDI and MGDI are for Henry Hub of Louisiana-Onshore South. We have

$$\Pi = V \sum_i \Theta\left[T_{avg}(i) - T_K\right] \max\left\{\mathrm{DGDI}(i) - \mathrm{MGDI}, 0\right\}$$

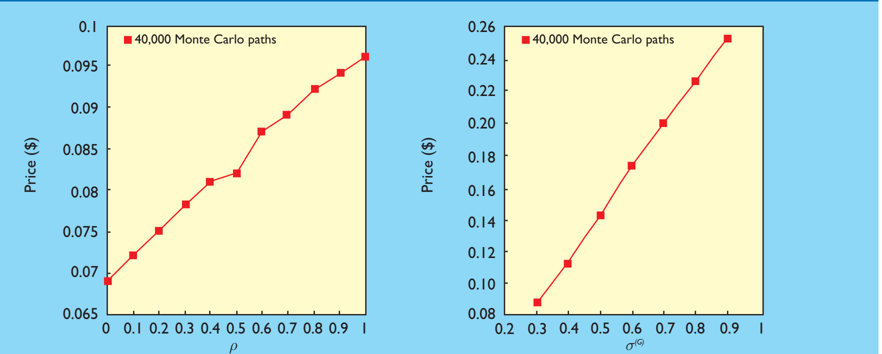
where i runs over the days in August. Θ = 1 for positive arguments and 0 otherwise. On 11 April 2003, the DGDI is \$5 and the risk-free interest rate is 1%.

First, we calibrate the models by using the quotes available in the natural gas and weather markets. The market price of the natural gas option without the weather trigger is \$0.36 per day and unit volume. The options market for weather derivatives implies a probability of the weather event $T_{avg} \geq 84$ of approximately 20%. For our choice of the underlying model dynamics, these values give the estimates $\sigma^{(G)}$ = 30% and $\sigma^{(T)}$ equal to 1.5 times the 10-year historical volatility.

Different models would imply different values of the volatilities. For example, the same market price for the natural gas option without the weather trigger can be obtained by use of a jump-diffusion model with a lower volatility and few jumps per year[3]. After the calibration is complete, we can run the simulation of a two-dimensional stochastic process with only one degree of freedom, ρ. Before proceeding further, it is worth mentioning that each numerical simulation in this paper has been done with 40,000 antithetic Monte Carlo paths.

In the left panel of Figure 1 we show how the price $\wp$ of the double-trigger weather vs natural gas call option depends upon the correlation coefficient ρ. As expected, we find that for ρ = 0 (ie, when weather and gas are driven by two independent Wiener processes) the price reduces to the one of the natural gas call option \$0.36 multiplied by the mar-

1. Price of a double-trigger weather vs natural gas call option plotted against the correlation coefficient ρ (left panel) and the volatility $\sigma^{(G)}$ (right panel)



1 N Ernst, 'Bringing it all together', *Environmental Finance*, February 2003, page 28.

2 See, for example, F Dornier, M Queruel, 'Caution to the wind', *EPRM*, August 2000, page 30.

3 R Carmona, D Villani, *Weather Derivatives*, Princeton University Press (forthcoming, 2004).

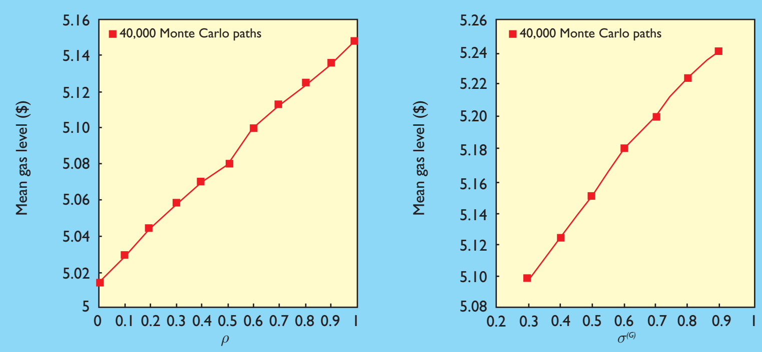
ket probability 20% of weather triggers (~\$0.07). This is a lower bound for the double-trigger option. One obvious, but not very constraining, upper bound is given by the price of the natural gas option \$0.36. For positive correlations, $\wp$ is an increasing function of ρ with an upper bound at $\rho = 1$ that is 40% larger than the value at $\rho = 0$ (an identical analysis can be carried out for negative correlations during the winter season).

So, if all the other parameters (including the volatilities) are held fixed, the price is in one-to-one correspondence with the correlation coefficient ρ, and prices can be quoted in correlation coefficient units, in the same way European call and put options are priced in terms of their implied volatilities.

If, at this point, a market player believes that a fair price for the double-trigger option needs to be in the range \$0.069–0.096 (see left panel of Figure 1), it might be assumed that s/he would likely go short the double-trigger option if there was a bid in the market at \$0.1. Despite its obvious intuitive appeal, however, this type of analysis can be disastrous.

Even if the gas options market implies the volatility $\sigma^{(G)} = 30\%$, this is not a good estimate for the volatility during the days in which the weather triggers (ie, when it is very hot at NY LaGuardia International Airport). Without going into the technical aspects of conditional variances, it is only fair to use increased gas volatilities to price double-trigger options.

2. Mean level of gas price for $T_{avg} \geq 84$ plotted against the correlation coefficient ρ (left panel) and the volatility $\sigma^{(G)}$ (right panel)



This is not a detail, as we show in the right panel of Figure 2, where we fixed $\rho = 0.6$. In fact, as a function of the natural gas volatility, the price $\wp$ seems to grow at the rate of 0.3. In this respect, if $\rho = 0$ represents the lower bound (starting bid level) for the option, the upper bound (starting ask level) could easily be three to four times as much. In both panels of Figure 2 we show the same results in terms of the mean level of the natural gas price on the days for which the weather triggers. This is a complementary view where the price is not mapped on a value of the correlation but instead on the mean price of gas during hot days.

In conclusion, we have shown how to price weather-contingent options by use of Monte Carlo simulations. We have analysed in some detail the double-trigger weather vs natural gas call option. The correlation between the natural gas and weather markets emerged as a quoting device similar to the implied volatility of the Black–Scholes paradigm. Finally, it is worth pointing out that our approach gives an exact analytical formula in some limiting regimes: more work in this direction is in progress. EF

René Carmona is the Paul M Wythes '55 professor of engineering and finance at Princeton University. E-mail: rcarmona@princeton.edu.
Dario Villani is a commodity trader with Hess Energy Trading. E-mail: dvillani@hess.com