Predatory Trading

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Premises for Predatory Trading

- Large Trader facing a Forced Liquidation
- Especially if the need to liquidate is known by other traders
  - hedge funds with (nearing) margin call
  - traders who use portfolio insurance, stop loss orders, …
  - some institutions / funds cannot hold on to downgraded instruments
  - Index-replication funds (at re-balancing dates) e.g. Russell 3000

Forced liquidation can be very costly because of price impact

Business Week

...if lenders know that a hedge fund needs to sell something quickly, they will sell the same asset, driving the price down even faster. Goldman Sachs and other counter-parties to LTCM did exactly that in 1998.

When you smell blood in the water, you become a shark. . . . when you know that one of your number is in trouble. . . . you try to figure out what he owns and you start shorting those stocks. . . .

Cramer (2002)
Typical Predatory Trading Scenario

- Distressed trader (prey) needs to unload a large position
  - Size will have impact on price
- **Predator** initially trades in the same direction as the prey
  - Effect is to withdraw liquidity
  - Market impact of the liquidation becomes greater
  - Price fall is exaggerated (**over-shooting**)
- Predator **reverses direction**, profiting from the price over-shoot
- Predator **closes position** for a profit.

Brunnermeier - Pedersen (2005)
Carlin - Lobo - Viswanathan (2005)
Schied - Schöneborn (2008)
Multi-Player Game Model

- One risk free asset and one risky asset
- Trading in continuous time, interest rate $r = 0$
- $n + 1$ strategic players and a number of noise traders
- $X_0(t), X_1(t), \cdots, X_n(t)$ risky asset positions of the strategic players
- Trades at time $t$ are executed at the price (Chriss-Almgren price impact model)

$$P(t) = \tilde{P}(t) + \gamma \sum_{i=0}^{n} [X_i(t) - X_i(0)] + \lambda \sum_{i=0}^{n} \dot{X}_i(t)$$

where $\tilde{P}(t)$ is a mean zero martingale (say a Wiener process).
Goal of the Mathematical Analysis

- Understand predation
- Illustrate benefits of
  - Stealth trading
  - Sunshine trading

**Modeling extreme markets**

- **Elastic** markets:
  - temporary impact $\lambda >>$ permanent impact $\gamma$
- **Plastic** markets:
  - permanent impact $\gamma >>$ temporary impact $\lambda$
Assumptions of the One Period Game

- Each strategic player $i \in \{0, 1, \cdots, n\}$ knows
  - all other strategic players initial asset positions $X_j(0)$ for $j \neq i$
  - Their target $X_j(T)$ at some fixed time point $T > 0$ in the future
- Objective (all players are risk neutral)
  - Players maximize their expected return by choosing an optimal trading strategy $X_i(t)$ satisfying their constraints $X_i(0)$ and $X_i(T)$

One distressed trader / prey (e.g. seller), player 0

$$X_0(0) = x_0 > 0, \quad X_0(T) = 0$$

$n$ predators players $1, 2, \cdots, n$

$$X_i(0) = X_i(T) = 0, \quad i = 1, \cdots, n$$
Optimization Problem

A strategy $X_i = (X_i(t))_{0 \leq t \leq T}$ is **admissible** (for player $i$) if it is an a

- adapted process
- with continuously differentiable sample paths

Given a set $X = (X_0, X_1, \cdots, X_n)$ of admissible strategies

- Each player $i \in \{0, 1, \cdots, n\}$ tries to maximize his expected return

$$J^i(X) = \mathbb{E}\left[ \int_0^T (-\dot{X}_i(t))P(t)dt \right]$$

under the constraint

$$P(t) = \tilde{P}(t) + \gamma \sum_{i=0}^{n} [X_i(t) - X_i(0)] + \lambda \sum_{i=0}^{n} \dot{X}_i(t)$$

- Search for **Nash Equilibrium**
Deterministic Strategies

If we restrict the admissible strategies $X = (X_0, X_1, \cdots, X_n)$ to be DETERMINISTIC

$$J^i(X) = \mathbb{E}\left[\int_0^T (-\dot{X}_i(t))P(t)dt\right] = \int_0^T (-\dot{X}_i(t))\bar{P}(t)dt$$

where

$$\bar{P}(t) = P(0) + \gamma \sum_{i=0}^{n} [X_i(t) - X_i(0)] + \lambda \sum_{i=0}^{n} \dot{X}_i(t)$$

THE SOURCE OF RANDOMNESS IS GONE!

Solution in the Deterministic Case

Unique Optimal Strategies

\[
X_i(t) = a e^{-\frac{n}{n+2} \frac{\gamma}{\lambda} t} + b_i e^{\frac{\gamma}{\lambda} t}
\]

where

\[
a = \frac{n}{n+2} \frac{\gamma}{\lambda} \left(1 - e^{-\frac{n}{n+2} \frac{\gamma}{\lambda} T}\right)^{-1} \left[\frac{1}{n+1} \sum_{i=0}^{n} [X_i(T) - X_i(0)] \right]
\]

\[
b_i = \frac{\gamma}{\lambda} \left( e^{\frac{\gamma}{\lambda} T} - 1 \right)^{-1} \left( X_i(T) - X_i(0) - \frac{1}{n+1} \sum_{i=0}^{n} [X_i(T) - X_i(0)] \right)
\]

Carlin - Lobo - Viswanathan (2005)
$n = 1$ predator, $\gamma/\lambda = 0.3$
$n = 1$ predator, $\gamma = \lambda$
$n = 1$ predator, $\gamma = 15.5\lambda$
Holdings of the Distressed Trader & Predator

![Graph showing the holdings of the Distressed Trader & Predator over time. The graph has a linear trending blue line and a horizontal red line.](image-url)
Fancy Plots of the Holdings of the Distressed Trader & Predator

Holdings of Distressed Trader (black) & Predator (red)
Impact of the Number of Predators: $\gamma = \lambda$

Holdings of Distressed Trader & 1 Predator

Holdings of Distressed Trader & 50 Predators
Impact of the Number of Predators: $\gamma = 15.5 \lambda$
Expected Price: $\gamma = \lambda$
Expected Price: $\gamma = 15\lambda$
Impact of Nb of Predators on Expected Returns

Expected Returns of Distressed Trader $GOL=1$ & $GOL=15$

Expected Returns of Predators $GOL=1$ & $GOL=15$
Two Period Model

- Prey has to liquidate $X_0 > 0$ by time $T_1$, i.e. $X_0(T_1) = 0$
- Predators can stay in the game longer $X_i(0) = X_i(T_2) = 0$ for some $T_2 > T_1$ for $i = 1, \ldots, n$
- Prey does not trade in second period $[T_1, T_2]$, i.e. $X_0(t) = 0$ for $T_1 \leq t \leq T_2$.

Markovian Structure $\implies$

**Solution determined by predators’ positions at time $T_1$**
Nash Equilibrium for Deterministic Strategies

**UNIQUE Nash Equilibrium**

- **ALL** Predators have the same position at time $T_1$

$$X_i(T_1) = \frac{A_2 n^2 + A_1 n + A_0}{B_3 n^3 + B_2 n^2 + B_1 n + B_0} X_0, \quad i = 1, \ldots, n$$

- Coefficients depend upon $n$ but converge as $n \to \infty$
- Asymptotic formulas for expected returns
- Asymptotic comparison of **Stealth** versus Sunshine trading for some regimes of $\gamma / \lambda$

_Schöneborn - Schied (2008)_