Plan of the Course

- **Commodity Markets**
  - Production, Transportation, Storage, Delivery
  - Spot / Forward Markets

- **Spread Option Valuation**
  - Why Spread Options
  - First Asset Valuation

- **Gas and Power Markets**
  - Physical / Financial Contracts
  - Price Formation
  - Load and Temperature

- **Weather Markets**
  - Weather Exposure
  - Temperature Options

- **More Asset Valuation**
  - Plant Optionality Valuation
  - Financial Valuation
  - Valuing Storage Facilities

- **Emission Markets**
Deregulated Electricity Markets

No More **Utilities monopolies**

Vertical Integration of *production, transportation, distribution* of electricity

**Unbundling**

Open competitive markets for production and retail
(Typically, grid remains under control)

**New Price Formation**

Constant *supply - demand* balance (Market forces)
Commodities form a **separate asset class**!

**LOCAL STACK – MERIT ORDER** (plant on the margin)
Support portfolio management
(producer, retailer, utility, investment banks, ...)

- Different **data analysis**
  (spot, day-ahead, on-peak, off-peak, firm, non-firm, forward, ... , negative prices)

- New instrument **valuation**
  (swing / recall / take-or-pay options, weather and credit derivatives, gas storage, cross commodity derivatives, ...)

- New forms of **hedging** using physical assets
  Perfected by **GS & MS** (power plants, pipelines, tankers, ...)

- Marking to market and new forms of **risk** measures
Degradation of credit exacerbated liquidity problems

- **Credit risk**
  - Understanding the statistics of credit migration
  - Including counter-party risk in valuation
  - Credit derivatives and credit enhancement

- **Reporting** and indexes

- Could **clearing** be a solution?
  - Exchange traded instruments pretty much standardized, but OTC!
  - Design of a minimal set of instruments for **standardization**

- **Collateral** requirements / **margin** calls
  - **Objective valuation** algorithms widely accepted for frequent Mark-to-Market
  - **Netting**
    - Challenge of the dependencies (correlations, copulas, ....)
    - Integrated approach to risk control
Physical Markets
- Spot (immediate delivery) Markets
- Forward Markets

Volume Explosion with Financially Settled Contracts
- Physical / Financial Contracts
- Exchanges serve as Clearing Houses
- Speculators provide Liquidity

In IB, part of Fixed Income Desk

Seasonality / Storage / Convenience Yield
First Challenge: Constructing Forward Curves

- How can it be a challenge?
  - Just do a PCA!
    - "OK" for Crude Oil (backwardation/contango → 3 factors)
    - Not settled for Gas
    - Does not work for Electricity
  - Extreme **complexity** & **size** of the data (location, grade, peak/off peak, firm/non firm, interruptible, swings, etc)
  - Incomplete and inconsistent sources of information
  - **Liquidity** and wide Bid-Ask spreads (**smoothing**)
  - **Length** of the curve (**extrapolation**)

- Dynamic models **à la HJM**:
Crude Oil

Crude Oil-Brent 1Mth Fwd FOB U$/BBL before Katrina

More Crude Oil Data

Crude Oil-Brent 1Mth Fwd FOB U$/BBL

[Graph showing the price trend of Brent crude oil from 1994 to 2007]
Crude Oil Spot Volatility
Is the Forward the Expected Value of Future Spots?
Examples of Crude Oil Forward Curves

- Backwardation
- Contengo
In financial models where one can hold positions at no cost

\[ F(t, T) = S(t)e^{r(T-t)} \]

by a simple **cash & carry arbitrage** argument. In particular

\[ F(t, T) = \mathbb{E}\{S(T) \mid \mathcal{F}_t\} \]

for risk neutral expectations.

**Perfect Price Discovery**

In general (theory of normal **backwardation**)
- \( F(t, T) \) is a **downward biased** estimate of \( S(T) \)
- Spot price exceeds the forward prices
**Forward Price** = (risk neutral) conditional expectation of future values of **Spot Price**

- No **cash & carry** arbitrage argument
  - Is the spot really tradable?
  - What are its dynamics?
  - How do we *risk-adjust* them?

- **Convenience Yield** for storable commodities
  - Natural Gas, Crude Oil, . . .
  - Correct interest rate to compute present values
  - Does not apply to Electricity
For **storabe** commodities (still same cash & carry arbitrage argument)

\[ F(t, T) = S(t)e^{(r-\delta)(T-t)} \]

for \( \delta \geq 0 \) called **convenience yield.** (NOT FOR ELECTRICITY !)

Decompose \( \delta = \delta_1 - c \) with

- \( \delta_1 \) benefit from owning the physical commodity
- \( c \) cost of storage

Then

\[ f(t, T) = e^{r(T-t)}e^{-\delta_1(T-t)}e^{-c(T-t)} \]

- \( e^{r(T-t)} \) cost of **financing** the purchase
- \( e^{c(T-t)} \) cost of **storage**
- \( e^{-\delta_1(T-t)} \) sheer **benefit from owning** the physical commodity
Backwardation / Contango Duality

**Backwardation**
- \( T \leftarrow F(t, T) = S(t) e^{(r+c-\delta_1)(T-t)} \) decreasing if \( r + c < \delta_1 \)
  - Low cost of storage
  - Low interest rate
  - High benefit in holding the commodity

**Contango**
- \( T \leftarrow F(t, T) = S(t) e^{(r+c-\delta_1)(T-t)} \) increasing if \( r + c \geq \delta_1 \)
Natural Gas
Commodity Convenience Yield Models

**Gibson-Schwartz** Two-factor model
- $S_t$ commodity spot price
- $\delta_t$ convenience yield

**Risk Neutral Dynamics**

$$dS_t = \left( r_t - \delta_t \right) S_t \, dt + \sigma S_t \, dW^1_t,$$

$$d\delta_t = \kappa(\theta - \delta_t) \, dt + \sigma_\delta \, dW^2_t$$

**Major Problems**
- Explicit formulae (exponential affine model)
- Convenience yield implied from forward contract prices
- Unstable & Inconsistent ([R.C.-M. Ludkovski](#))
Lack of Consistency

Exponential Affine Model

\[ F(t, T) = S_t e^{\int_t^T r_s ds} e^{B(t, T) \delta_t + A(t, T)} \]

where

\[ B(t, T) = \frac{e^{-\kappa(T-t)} - 1}{\kappa}, \]
\[ A(t, T) = \frac{\kappa \theta + \rho \sigma_s \gamma}{\kappa^2} (1 - e^{-\kappa(T-t)} - \kappa(T-t)) + \]
\[ + \frac{\gamma^2}{\kappa^3} (2\kappa(T-t) - 3 + 4e^{-\kappa(T-t)} - e^{-2\kappa(T-t)}). \]

- For each \( T \), one can imply \( \delta_t \) from \( F(t, T) \)
- Inconsistency in the implied \( \delta_t \)
- Ignores **Maturity Specific** effects
Crude Oil convenience yield implied by a 3 month futures contract (left)
Difference in implied convenience yields between 3 and 12 month contracts.
Use **forward** $F_t = F(t, T_0)$ instead of **spot** $S_t$ ($T_0$ fixed maturity)

**Historical Dynamics**

\[
\begin{align*}
  dF_t &= (\mu_t - \delta_t)F_t\,dt + \sigma F_t\,dW_t^1, \\
  d\delta_t &= \kappa(\theta - \delta_t)\,dt + \sigma_\delta\,dW_t^2
\end{align*}
\]

or more generally

\[d\delta_t = b(\delta_t, F_t)\,dt + \sigma_\delta(\delta_t, F_t)dW_t^2\]

We assume

- $F_t$ is **tradable** (hence **observable**)
- (Forward) convenience yield $\delta_t$ **not observable** (filtering)

Different from **Bjork-Landen**'s **Risk Neutral Term Structure of Convenience Yield**
Several obstructions

- Cannot store the physical commodity
- Does the forward price converge as the time to maturity goes to 0?

Mathematical spot?

\[ S(t) = \lim_{T \downarrow t} F(t, T) \]

Sparse Forward Data

- Lack of transparency (manipulated indexes)
- Poor (or lack of) reporting by fear of law suits
- CCRO white paper(s)
Dynamic Model for Forward Curves

\[ n\text{-factor forward curve model} \]

\[
\frac{dF(t, T)}{F(t, T)} = \mu(t, T)dt + \sum_{k=1}^{n} \sigma_k(t, T)dW_k(t) \quad t \leq T
\]

- \( W = (W_1, \ldots, W_n) \) is a \( n \)-dimensional standard Brownian motion,
- drift \( \mu \) and volatilities \( \sigma_k \) are deterministic functions of \( t \) and time-of-maturity \( T \)
- \( \mu(t, T) \equiv 0 \) for pricing
- \( \mu(t, T) \) calibrated to historical data for risk management
Explicit Solution

\[ F(t, T) = F(0, T) \exp \left[ \int_0^t \left[ \mu(s, T) - \frac{1}{2} \sum_{k=1}^n \sigma_k(s, T)^2 \right] ds + \sum_{k=1}^n \int_0^t \sigma_k(s, T) dW_k(s) \right] \]

Forward prices are **log-normal** (deterministic coefficients)

\[ F(t, T) = \alpha e^{\beta X - \beta^2 / 2} \]

with \( X \sim N(0, 1) \) and

\[ \alpha = F(0, T) \exp \left[ \int_0^t \mu(s, T) ds \right], \quad \text{and} \quad \beta = \sqrt{\sum_{k=1}^n \int_0^t \sigma_k(s, T)^2 ds} \]
Dynamics of the Spot Price

**Spot price** left hand of forward curve

\[ S(t) = F(t, t) \]

We get

\[ S(t) = F(0, t) \exp \left[ \int_0^t [\mu(s, t) - \frac{1}{2} \sum_{k=1}^n \sigma_k(s, t)^2] ds + \sum_{k=1}^n \int_0^t \sigma_k(s, t) dW_k(s) \right] \]

and differentiating both sides we get:

\[
\begin{align*}
dS(t) &= S(t) \left[ \left( \frac{1}{F(0, t)} \frac{\partial F(0, t)}{\partial t} + \mu(t, t) + \int_0^t \frac{\partial \mu(s, t)}{\partial t} ds - \frac{1}{2} \sigma_S(t)^2 \right) \\
&\quad - \sum_{k=1}^n \int_0^t \sigma_k(s, t) \frac{\partial \sigma_k(s, t)}{\partial t} ds + \sum_{k=1}^n \int_0^t \frac{\partial \sigma_k(s, t)}{\partial t} dW_k(s) \right) dt + \sum_{k=1}^n \sigma_k(t, t) dW_k(t) \right]\end{align*}
\]

**Spot volatility**

\[
\sigma_S(t)^2 = \sum_{k=1}^n \sigma_k(t, t)^2. \quad (1)
\]
Hence

\[
\frac{dS(t)}{S(t)} = \left[ \frac{\partial \log F(0, t)}{\partial t} + D(t) \right] dt + \sum_{k=1}^{n} \sigma_k(t, t)dW_k(t)
\]

with drift

\[
D(t) = \mu(t, t) - \frac{1}{2} \sigma_S(t)^2 + \int_{0}^{t} \frac{\partial \mu(s, t)}{\partial t} ds - \sum_{k=1}^{n} \int_{0}^{t} \sigma_k(s, t) \frac{\partial \sigma_k(s, t)}{\partial t} ds
\]

\[
+ \sum_{k=1}^{n} \int_{0}^{t} \frac{\partial \sigma_k(s, t)}{\partial t} dW_k(s)
\]
Remarks

- Interpretation of drift (in a risk-neutral setting)
  - logarithmic derivative of the forward can be interpreted as a discount rate (i.e., the running interest rate)
  - \( D(t) \) can be interpreted as a convenience yield
- Drift generally not Markovian
- Particular case \( n = 1, \mu(t, T) \equiv 0, \sigma_1(t, T) = \sigma e^{-\lambda(T-t)} \)

\[
D(t) = \lambda [\log F(0, t) - \log S(t)] + \frac{\sigma^2}{4} \left(1 - e^{-2\lambda t}\right)
\]

\[
\frac{dS(t)}{S(t)} = [\mu(t) - \lambda \log S(t)] dt + \sigma dW(t)
\]

exponential OU
Changing Variables

\[ \text{time-of-maturity } T \quad \Rightarrow \quad \text{time-to-maturity } \tau \]

changes dependence upon \( t \)

\[ t \leftrightarrow F(t, T) = F(t, t + \tau) = \tilde{F}(t, \tau) \]

**Fixed Domain** \([0, \infty)\) for \( \tau \leftrightarrow \tilde{F}(t, \tau) \)
Heating Oil Forward Surface
HO PCA Loadings
HO Loadings on their Importance Scale
PCA of Heating Oil Forwards

Comp.1: 0.931
Comp.2: 0.979
Comp.3: 0.99
Comp.4: 0.997
Comp.5: 0.999
Comp.6: 1
Comp.7: 1
Comp.8: 1
Comp.9: 1
Comp.10: 1
HO Loadings on their Importance Scale

0.20
0.15
0.10
0.05
0.0
0 10 20 30
PCA of Henry Hub Natural Gas Forward Prices

0.922

Comp.1  Comp.2  Comp.3  Comp.4  Comp.5  Comp.6  Comp.7  Comp.8  Comp.9  Comp.10
Changing Variables

\[ \text{time-of-maturity } T \quad \Rightarrow \quad \text{time-to-maturity } \tau \]

changes dependence upon \( t \)

\[ t \hookrightarrow F(t, T) = F(t, t + \tau) = \tilde{F}(t, \tau) \]

For **pricing purposes**

- For \( T \) fixed, \( \{F(t, T)\}_{0 \leq t \leq T} \) **is a martingale**
- For \( \tau \) fixed, \( \{\tilde{F}(t, \tau)\}_{0 \leq t} \) **is NOT a martingale**

\[ \tilde{F}(t, \tau) = F(t, t+\tau), \quad \tilde{\mu}(t, \tau) = \mu(t, t+\tau), \quad \text{and} \quad \tilde{\sigma}_k(t, \tau) = \sigma_k(t, t+\tau), \]

In general dynamics become

\[ d\tilde{F}(t, \tau) = \tilde{F}(t, \tau) \left[ \left( \tilde{\mu}(t, \tau) + \frac{\partial}{\partial \tau} \log \tilde{F}(t, \tau) \right) dt + \sum_{k=1}^{n} \tilde{\sigma}_k(t, \tau) dW_k(t) \right], \quad \tau \geq 0. \]
PCA with Seasonality

Fundamental Assumption

\[
\sigma_k(t, T) = \sigma(t)\sigma_k(T - t) = \sigma(t)\sigma_k(\tau)
\]
for some function \( t \mapsto \sigma(t) \)

Notice

\[
\sigma_S(t) = \tilde{\sigma}(0)\sigma(t)
\]

provided we set:

\[
\tilde{\sigma}(\tau) = \sqrt{\sum_{k=1}^{n} \sigma_k(\tau)^2}.
\]

Conclusion

\( t \mapsto \sigma(t) \) is (up to a constant) the **instantaneous spot volatility**
Rationale for a New PCA

- Fix times-to-maturity $\tau_1, \tau_2, \ldots, \tau_N$
- Assume on each day $t$, quotes for the forward prices with times-of-maturity $T_1 = t + \tau_1$, $T_2 = t + \tau_2$, $\ldots$, $T_N = t + \tau_N$ are available

$$
\frac{d\tilde{F}(t, \tau_i)}{\tilde{F}(t, \tau_i)} = \left( \tilde{\mu}(t, \tau_i) + \frac{\partial}{\partial \tau} \log \tilde{F}(t, \tau_i) \right) dt + \sigma(t) \sum_{k=1}^{n} \sigma_k(\tau_i) dW_k(t) \quad i = 1, \ldots, N
$$

Define $F = [\sigma_k(\tau_i)]_{i=1,\ldots,N, k=1,\ldots,n}$.

$$
d \log \tilde{F}(t, \tau_i) = \left( \tilde{\mu}(t, \tau_i) + \frac{\partial}{\partial \tau_i} \log \tilde{F}(t, \tau_i) - \frac{1}{2} \sigma(t)^2 \tilde{\sigma}(\tau_i)^2 \right) dt + \sigma(t) \sum_{k=1}^{n} \sigma_k(\tau_i) dW_k(t),
$$

Instantaneous variance/covariance matrix $\{M(t); t \geq 0\}$ defined by:

$$
d[\log \tilde{F}(\cdot, \tau_i), \log \tilde{F}(\cdot, \tau_j)]_t = M_{i,j}(t) dt
$$

satisfies

$$
M(t) = \sigma(t)^2 \left( \sum_{k=1}^{n} \sigma_k(\tau_i) \sigma_k(\tau_j) \right)
$$

or equivalently

$$
M(t) = \sigma(t)^2 FF^*
$$
Estimate instantaneous spot volatility $\sigma(t)$ (in a rolling window)

Estimate $\mathbf{F}F^*$ from historical data as the empirical auto-covariance of $\ln(F(t, \cdot)) - \ln(F(t - 1, \cdot))$ after normalization by $\sigma(t)$

Instantaneous auto-covariance structure of the entire forward curve becomes time independent

Do SVD of auto-covariance matrix and get

$$\tau \leftrightarrow \sigma_k(\tau)$$

Choose order $n$ of the model from their relative sizes
The Case of Natural Gas

Instantaneous standard deviation of the Henry Hub natural gas spot price computed in a sliding window of length 30 days.
PCA of Henry Hub Natural Gas \( \Delta \)e-Seasonalized Forward Prices

Comp.1: 0.715
Comp.2: 0.902
Comp.3: 0.94
Comp.4: 0.956
Comp.5: 0.965
Comp.6: 0.972
Comp.7: 0.978
Comp.8: 0.982
Comp.9: 0.985
Comp.10: 0.988
HH De-Seasonalized Loadings on their Absolute Importance Scale
Demand, Risk Neutral Firms & Price Formation

- **Finite set** $\mathcal{I}$ of **risk neutral agents/firms**
- **Producing a finite set** $\mathcal{K}$ of **goods**
- Firm $i \in \mathcal{I}$ can use **technology** $j \in \mathcal{J}^{i,k}$ to produce good $k \in \mathcal{K}$
- **Discrete time** $\{0, 1, \cdots, T\}$
- **Demand for Goods**

\[ \{D^k(t); \ t = 0, 1, \cdots, T - 1, \ k \in \mathcal{K}\}. \]

- **Production Capacity Limits** $\kappa^{i,j,k} \geq 0$
Goal of Equilibrium Analysis

Find a stochastic process
- for the Prices of goods

\[ S = \{ S_t^k \}, k \in K, t \geq 0 \]

satisfying the usual conditions for the existence of a

competitive equilibrium
Individual Firm Problem

- If price of goods $S$ given exogenously
- If firm $i \in \mathcal{I}$ produces $\xi_{t}^{i,j,k}$ of good $k \in \mathcal{K}$ with technology $j \in \mathcal{J}^{i,k}$ during time period $[t, t + 1)$

then P&L of firm $i$ given by

$$L^{S,i}(\xi^{i}) := \sum_{k \in \mathcal{K}} \sum_{j \in \mathcal{J}^{i,k}} \sum_{t=0}^{T-1} (S^{k}_{t} - C_{t}^{i,j,k})\xi_{t}^{i,j,k}$$

Problem for (risk neutral) firm $i \in \mathcal{I}$

$$\max_{\xi^{i}, \ 0 \leq \xi_{t}^{i,j,k} \leq \kappa_{i,j,k}} \mathbb{E}\{L^{S,i}(\xi^{i})\}$$
Classical competitive equilibrium problem!

**Representative Agent / Informed Central Planner**

chooses optimal **production schedules** and the equilibrium prices $S^*$ are set so that supply meets demand. For each time $t$

$$(\xi^*_{t,i,j,k})_{i,j,k} = \arg\max_{((\xi_{t,i,j,k})_{i,j,k})_{i\in I, j\in J, k}} \sum_{i\in I} \sum_{j\in J} \sum_{k\in K} -C_{t,i,j,k} \xi_{t,i,j,k}$$

$$\sum_{i\in I} \sum_{j\in J} \xi_{t,i,j,k} = D_{t,k} \quad k \in K$$

$$0 \leq \xi_{t,i,j,k} \leq \kappa_{i,j,k} \quad \text{for } i \in I, j \in J, k \in K$$
Classical competitive equilibrium problem!

**Representative Agent / Informed Central Planner**

chooses optimal **production schedules** and the equilibrium prices $S^*$ are set so that supply meets demand. For each time $t$

\[
(\xi^*_{i,j,k})_{i,j,k} = \arg \max_{((\xi^i_{i,j,k})_{i,j,k})_{i \in I, j \in J_i, k}} \sum_{i \in I} \sum_{j \in J_i, k} -C^i_{i,j,k} \xi^i_{i,j,k}
\]

\[
\sum_{i \in I} \sum_{j \in J_i, k} \xi^i_{i,j,k} = D^k_t \quad k \in K
\]

\[
0 \leq \xi^i_{i,j,k} \leq \kappa^i_{i,j,k} \quad \text{for } i \in I, j \in J_i, k \in K
\]
The corresponding prices of the goods are

\[ S_{t}^{*k} = \max_{i \in I, j \in J^{i,k}} C_{t}^{i,j,k} 1_{\{\xi_{t}^{*i,j,k} > 0\}} \]

**Classical MERIT ORDER**

- At each time \( t \) and for each good \( k \)
- Production technologies ranked by increasing production costs \( C_{t}^{i,j,k} \)
- Demand \( D_{t}^{k} \) met by producing from the cheapest technology first
- Equilibrium spot price is the marginal cost of production of the most expansive production technology used to meet demand

**Business As Usual**
(typical scenario in Deregulated electricity markets)
The corresponding prices of the goods are

\[ S_t^{*k} = \max_{i \in I, j \in J^i, k} C_t^{i,j,k} 1_{\{\xi_t^{*i,j,k} > 0\}} , \]

**Classical MERIT ORDER**

- At each time \( t \) and for each good \( k \)
- Production technologies ranked by increasing production costs \( C_t^{i,j,k} \)
- Demand \( D_t^k \) met by producing from the cheapest technology first
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**Business As Usual**
(typical scenario in Deregulated electricity markets)
Reduced Form Models

Based on idea that

"Commodities **Mean Revert**" toward the **cost of production**

Case of power prices

- **Models for ”Spot” Price**
  - Nonlinear effects (exponential $OU^2$)
  - Jumps diffusion models

- **Structural Models**
  - Inelastic Demand $\rightarrow$ Supply Stack & **Merit Order**

**Barlow**

- $s_t(x)$ supply at time $t$ when power price is $x$
- $d_t(x)$ demand at time $t$ when power price is $x$

**Power price** at time $t$ is number $S_t$ such that

$$s(S_t) = d_t(S_t)$$
Example of a merit graph (Alberta Power Pool, courtesy M. Barlow)
Barlow’s Proposal for a Dynamic Model

Same supply every day

\[ s_t(x) = g(x) \]

Inelastic demand

\[ d_t(x) = D_t \]

So

\[ S_t = g^{-1}(D_t) = f(D_t) \]

Barlow chooses

\[ S_t = \begin{cases} f_\alpha(X_t) & 1 + \alpha X_t > \epsilon_0 \\ \epsilon_0^{1/\alpha} & 1 + \alpha X_t \leq \epsilon_0 \end{cases} \]

for the non-linear function, including a "cut-off",

\[ f_\alpha(x) = \begin{cases} (1 + \alpha x)^{1/\alpha}, & \alpha \neq 0 \\ e^x & \alpha = 0 \end{cases} \]

of an OU diffusion

\[ dX_t = -\lambda(X_t - \bar{x})dt + \sigma dW_t \]
Monte Carlo Sample from Barlow’s Spot Model (courtesy M. Barlow)
Example of a Monte Carlo Sample from the Exponential of an $OU^2$
Consider the case of **PJM** (Pennsylvania - New Jersey - Maryland)

- Over 3,000 nodes in the transmission network
- Each day, and for each node
  - Real time prices
  - Day-ahead prices
  - Hour by hour load prediction for the following day

**Historical prices**

- In 2003 over 100,000 instances of **NEGATIVE PRICES**
  - Geographic clusters
  - Time of the year (*shoulder months*)
  - Time of the day (*night*)

**Possible Explanations**

- Load miss-predicted
- High temperature volatility
For many contracts, delivery needs to match demand

- **Demand** for energy highly correlated with **temperature**
  - Heating Season (winter) HDD
  - Cooling Season (summer) CDD
- **Stylized Facts** and **First (naive) Models**
  - Electricity demand $= \beta \times \text{weather} + \alpha$
Daily Load versus Daily Temperature (PJM)
For many contracts, delivery needs to match demand

- **Demand** for energy highly correlated with **temperature**
  - Heating Season (winter) HDD
  - Cooling Season (summer) CDD

- **Stylized Facts** and **First (naive) Models**
  - Electricity demand = $\beta \times$ weather + $\alpha$
    - Not true all the time
    - Time dependent $\beta$ by filtering!
  - From the stack: Correlation (Gas, Power) = $f($weather$)$
    - No significance, too unstable
    - Could it be because of heavy tails?

- **Weather dynamics** need to be included
  - **Another Source of Incompleteness**
Princeton University Electricity Budget

2.8 M $ over (PU is small)

- The University has its own Power Plant
- Gas Turbine for Electricity & Steam

Major Exposures
- Hot Summer (air conditioning) Spikes in Demand, Gas & Electricity Prices
- Cold Winter (heating) Spikes in Gas Prices
Never Again such a Short Fall !!!

Student (Greg Larkin) Senior Thesis

**Hedging Volume Risk**
- Protection against the Weather Exposure
- *Temperature Options* on CDDs (Extreme Load)

**Hedging Volume & Basis Risk**
- Protection against Gas & Electricity Price Spikes
- Gas purchase with *Swing Options*
Mitigating Volume Risk with Swing Options

Exposure to spikes in prices of
- Natural Gas (used to fuel the plant)
- Electricity Spot (in case of overload)

Proposed Solution
- Forward Contracts
- Swing Options

Pretty standard
Mitigating Volume Risk

- Use **Swing Options**
- Multiple Rights to deviate (within bounds) from base load contract level
- **Pricing & Hedging** quite involved!
  - Tree/Forest Based Methods
    - Direct Backward Dynamic Programing Induction
      (à la Jaillet-Ronn-Tompaidis)
  - **New Monte Carlo Methods**
    - Nonparametric Regression (à la Longstaff-Schwarz) Backward Dynamic Programing Induction
Review: **Classical Optimal Stopping Problem: American Option**

- $X_0, X_1, X_2, \ldots, X_n, \ldots$ rewards
- Right to ONE Exercise
- Mathematical Problem

\[
\sup_{0 \leq \tau \leq T} \mathbb{E}\{X_\tau\}
\]

**Mathematical Solution**

- Snell’s Envelop
- Backward Dynamic Programming Induction in Markovian Case

*Standard, Well Understood*
In its simplest form the problem of Swing/Recall option pricing is an **Optimal Multiple Stopping Problem**

- $X_0, X_1, X_2, \ldots, X_n, \ldots$ rewards
- Right to $N$ Exercises
- Mathematical Problem

$$\sup_{0 \leq \tau_1 < \tau_2 < \cdots < \tau_N \leq T} \mathbb{E}\{X_{\tau_1} + X_{\tau_2} + \cdots + X_{\tau_N}\}$$

- **Refraction** period $\theta$

$$\tau_1 + \theta < \tau_2 < \tau_2 + \theta < \tau_3 < \cdots < \tau_{N-1} + \theta < \tau_N$$

Part of recall contracts & crucial for continuous time models
Instruments with Multiple American Exercises

- **Ubiquitous in Energy Sector**
  - Swing / Recall contracts
  - End user contracts (EDF)
- **Present in other contexts**
  - Fixed income markets (e.g. chooser swaps)
  - Executive option programs
    - Reload → Multiple exercise, Vesting → Refraction, ⋯
  - Fleet Purchase (airplanes, cars, ⋯)
- **Challenges**
  - Valuation
  - Optimal exercise policies
  - Hedging
Some Mathematical Problems

Recursive re-formulation into a hierarchy of classical optimal stopping problems

- Development of a theory of *Generalized Snell’s Envelop* in continuous time setting
- Find a form of Backward Dynamic Programing Induction in Markovian Case
- Design & implement efficient numerical algorithms for finite horizon case

Results

- Perpetual case: abstract nonsense
  R.C. & S.Dayanik (diffusion), R.C. & N.Touzi (GBM)
- Perpetual case: Characterization of the optimal policies
  R.C. & S.Dayanik (diffusion), R.C. & N.Touzi (GBM)
- Finite horizon case
  Jaillet - Ronn - Tomapidis (Tree) R.C. N.Touzi (GBM) B.Hambly (chooser swap)
Exercise regions for $N = 5$ rights and finite maturity computed by Malliavin-Monte-Carlo.
Mitigation of Volume Risk with Temperature Options

- Rigorous Analysis of the Dependence between the **Budget Shortfall** and **Temperature** in Princeton

- Use of Historical Data (**sparse**) & Define of a **Temperature Protection**
  - Period of the Coverage
  - Form of the Coverage

- Search for the **Nearest Weather Stations** with HDD/CDD Trades
  - La Guardia Airport (LGA)
  - Philadelphia (PHL)

- Define a Portfolio of LGA & PHL forward / option Contracts

- Construct a **LGA / PHL basket**
Pricing: How Much is it Worth to PU?

- **Actuarial / Historical Approach**
  - Burn Analysis
  - Temperature Modeling & Monte Carlo VaR Computations
  - Not Enough Reliable Load Data

- **Expected (Exponential) Utility Maximization (A. Danilova)**
  - Use Gas & Power Contracts
  - Hedging in Incomplete Models
  - Indifference Pricing
  - Very Difficult Numerics (whether PDE’s or Monte Carlo)
Weather is an essential economic factor

- *Weather is not just an environmental issue; it is a major economic factor. At least 1 trillion USD of our economy is weather-sensitive* (William Daley, 1998, US Commerce Secretary)
- **20% of the world economy** is estimated to be affected by weather
- Energy and other industrial sectors, Entertainment and Tourism Industry, ...
- **WRMA**

Weather Derivatives as a **Risk Transfer** Mechanism (**El Karoui - Barrieu**)
Total Notional Value of weather contracts: (in million USD) Price Waterhouse Coopers market survey.
Weather Derivatives

- **OTC** Customer tailored transactions
  - Temperature, Precipitation, Wind, Snow Fall, ..... 
- **CME** (≈ 50%) (Temperature - Launched in 1999)
  - 18 American cities
  - 2 Japanese cities (Tokyo and Osaka)
An Example of Precipitation Contract

Physical Underlying Daily Index:
- Precipitation in Paris
- A day is a rainy day if precipitation exceeds 2mm

Season
- 2000: April thru August + September weekends
- 2001: April thru August + September weekends
- 2002: April thru August + September weekends

Aggregate Index
- Total Number of Rainy Days in the Season

Pay-Off
- Strike, Cap, Rate
Who Wanted this Deal?
- A Natural Trying to Hedge RainFall Exposure (Asterix Amusement Park)

Who was willing to take the other side?
- Speculators
- Insurance Companies
- Re-insurance Companies
- Statistical Arbitrageurs
- Investment Banks
- Hedge Funds
- Endowment Funds
- ..................
Other Example: Precipitation / Snow Pack

- City of Sacramento
  - HydroPower Electricity
- Who was on the other side?
  - Large Energy Companies (*Aquila, Enron*)

**Who is covering for them?**
For a given **location**, on any given day \( t \)

\[
CDD_t = \max\{ T_t - 65, 0\} \quad \text{and} \quad HDD_t = \max\{ 65 - T_t, 0\}
\]

**Season**
- One Month (CME Contracts)
- May 1st September 30 (CDD season)
- November 1st March 31st (HDD season)

**Index**
- Aggregate number of DD in the season

\[
l = \sum_{t \in \text{Season}} CDD_t \quad \text{or} \quad l = \sum_{t \in \text{Season}} HDD_t
\]

**Pay-Off**
- Strike \( K \), Cap \( C \), Rate \( \alpha \)
Call with Cap

Pay-off = \min\{\max\{\alpha \cdot (I - K), 0\}, C\}

\xi = f(DD)

\begin{align*}
\xi &= f(DD) \\
C &= \text{constant line}
\end{align*}
Put with a Floor

\[ \xi = f(DD) \]

\[ F \]

Pay-off = \( \min\{ \max\{ \alpha \times (K - l), 0 \}, C \} \)
Collar

\[ \xi = f(DD) \]

\[ C \]

\[ K_p \]

\[ K_c \]

\[ -F \]
Famous Example of Weather Station Change in Charlotte (NC).
**Structure:** Heating Degree Day (HDD) Floor (Put)

**Index:** Cumulative HDDs

**Term:** November 1, 2007  February 28, 2008

**Stations:**
- New York, LaGuardia  57.20%
- Boston, MA  24.5%
- Philadelphia, PA  12.00%
- Baltimore, MD  6.30%

**Floor Strike:** 3130 HDDs

**Payout:** USD 35,000/HDD

**Limit:** USD 12,500,000

**Premium:** USD 2,925,000
Weather and Commodity

- **Stand-alone**
  - temperature ($\approx 80\%$)
  - precipitation ($\approx 10\%$)
  - wind ($\approx 5\%$)
  - snow fall ($\approx 5\%$)

- **In-Combination**
  - natural gas
  - power
  - heating oil
  - propane

- Agricultural risk (yield, revenue, input hedges and trading)
- Power outage - contingent power price options
Still Extremely **Illiquid** Markets (except for **front month**)

**Misconception:** Weather Derivative = Insurance Contract
- No secondary market (Except on **Enron-on-Line!!!**)

**Mark-to-Market** (or Model)
- Essentially never changes
- At least, Not Until Meteorology kicks in (10-15 days before maturity)
- Then Mark-to-Market (or Model) changes every day
- Contracts change hands
- That’s when major losses occur and money is made

This *hot period* is not considered in academic studies
- Need for **updates:** new information coming in (temperatures, forecasts, ....)
- Filtering is (again) the solution
La Guardia Daily Average Temperature

Daily Average Temperature at La Guardia.
Prediction on 6/1/2001 of daily temperature over the next four months.
The Future of the Weather Markets

- **Social function** of the weather market
  - Existence of a Market of Professionals (for weather risk transfer)

- **Under attack** from
  - (Re-)Insurance industry (but *high frequency / low cost*)
  - Utilities (trying to pass weather risk to end-customer)
    - EDF program in France
    - Weather Normalization Agreements in US

- **Cross Commodity Products**
  - Gas & Power contracts with *weather triggers/contingencies*
  - New (major) players: **Hedge Funds** provide liquidity

- **World Bank**
  - Use weather derivatives instead of insurance contracts
The Weather Market Today

- **Insurance Companies**: Swiss Re, XL, Munich Re, Ren Re
- **Financial Houses**: Goldman Sachs, Deutsche Bank, Merrill Lynch, SocGen, ABN AMRO
- **Hedge funds**: D. E. Shaw, Tudor, Susquehanna, Centaurus, Wolverine

**Where is Trading Taking Place?**
- **Exchange**: CME (Chicago Mercantile Exchange) 29 cites globally traded, monthly / seasonal contracts
- **OTC**
- Strong end-user demand within the **energy sector**
Temperature Options: Actuarial/Statistical Approach
Temperature Options: Diffusion Models (Danilova)
Precipitation Options: Markov Models (Diko)

- **Problem:** Pricing in an Incomplete Market
- **Solution:** Indifference Pricing à la Davis

\[
\begin{align*}
    d\theta_t &= p(t, \theta)dt + q(t, \theta)dW_t^{(\theta)} + r(t, \theta)dQ_t^{(\theta)} \\
    dS_t &= S_t[\mu(t, \theta)dt + \sigma(t, \theta)dW_t^{(S)}]
\end{align*}
\]

- \( \theta_t \) **non-tradable**
- \( S_t \) **tradable**
Example: **Exponential Utility Function**

\[
\tilde{p}_t = \frac{\mathbb{E}\{\tilde{\phi}(Y_T) e^{-\int_t^T V(s, Y_s)ds}\}}{\mathbb{E}\{e^{-\int_t^T V(s, Y_s)ds}\}}
\]

where

- \(\tilde{\phi} = e^{-\gamma(1-\rho^2)t}\)

  where \(f(\theta_T)\) is the pay-off function of the European call on the temperature

- \(\tilde{p}_t = e^{-\gamma(1-\rho^2)p_t}\)

  where \(p_t\) is price of the option at time \(t\)

- \(Y_t\) is the diffusion:

\[
dY_t = [g(t, Y_t) - \frac{\mu(t, Y_t) - r}{\sigma(t, Y_t)} h(t, Y_t)] dt + h(t, Y_t) d\tilde{W}_t
\]

starting from \(Y_0 = y\)

- \(V\) is the time dependent potential function:

\[
V(t, y) = -\frac{1 - \rho^2}{2} \frac{(\mu(t, y) - r)^2}{\sigma(t, y)^2}
\]
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